Comment on "Intermittency exponent of the turbulent energy cascade"

B. Castaing
Ecole Normale Supérieure de Lyon, 46 Allée d'Italie, 69364 Lyon Cedex 7, France

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It is argued that the method used by Cleve *et al.* [Phys. Rev. E **69**, 066316 (2004)], to determine the intermittency exponent μ is biased at moderate Reynolds numbers R_λ. Thus, the claimed dependence of μ upon (R_λ) is questionable. Other determinations give a constant value for μ.

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In a recent paper [1], Cleve *et al.* aimed to determine the intermittency exponent μ through the scaling of the two-point correlation function $\langle\epsilon(x+d)\epsilon(x)\rangle/\langle\epsilon\rangle^2$ of the energy dissipation ϵ. They study its behaviour versus the Taylor scale based Reynolds number R_λ, analyzing three types of experiments: atmospheric boundary layer [2], a wind tunnel shear flow [3], and a gaseous helium jet flow [4]. Through their analysis, μ appears as depending on the Reynolds number R_λ, increasing up to $R_\lambda \simeq 10^3$, saturating only for the atmospheric data, with a clear discrepancy between the helium and wind tunnel data. Cleve *et al.* [1] suggest that noise in the helium data could be responsible for the discrepancy.

Here, we explain that the μ values obtained in [1] are biased for moderate R_λ, or large turbulence ratio. Our comment is threefold. First, we remark that the two points correlation function studied in [1] is equivalent to another one already studied by Delour *et al.* [5]. Second (which is the main point of the Comment) we explain that it is the failure of the Taylor hypothesis at low R_λ which introduces a bias [6]. Third, we point out that noise in the helium data is not a problem.

Reference [5] studied the two-point correlation of the logarithms of velocity increments $\delta_\ell(x)=v(x+\ell)-v(x)$:

$$C_{\ell\ell}(d) = \langle \ln[|\delta_\ell(x+d)|]\ln[|\delta_\ell(x)|]\rangle - \langle \ln(|\delta_\ell|)\rangle^2.$$

For two Gaussian random variables y and z, of zero mean: $\langle e^{y+z}\rangle=\langle e^y\rangle\langle e^z\rangle e^{\langle yz\rangle}$. As the amplitude of velocity increments is proportional to $\epsilon^{1/3}$ [7,8], $9C_{\ell\ell}(d)$ equals the logarithm of the Cleve *et al.*[1] two points ϵ correlation.

Delour *et al.* [5] found the correlation $C_{\ell\ell}(d)$ having a behavior different from the second cumulant C_2 of the logarithm of velocity increments, which also gives μ:

$$C_2(d) = \langle[\ln(|\delta_d|)]^2\rangle - \langle\ln(|\delta_d|)\rangle^2.$$

It has been shown in [4,5] that $C_2(d)=(\mu/9)\ln\left(\frac{L}{d}\right)$, up to the integral scale L. They found $\mu=0.22\pm0.03$, independent of R_λ, a value in full agreement with that of [1] for atmospheric data. The difference between $C_{\ell\ell}(d)$ and $C_2(d)$, in apparent contradiction with cascade models, has been shown in [6] to originate in the failure of the Taylor hypothesis of frozen turbulence. Indeed, the turbulence is really frozen only during the Kolmogorov time $\tau_\eta=(\nu/\epsilon)^{1/2}$, where ν is the kinematic viscosity. The corresponding distance is $d_m=U\tau_\eta\simeq\lambda U/(4U')$, U'/U being the turbulence ratio, and λ the Taylor scale. For distances larger than d_m, further temporal decorrelation occurs, resulting in a different behaviour, quadratic instead of linear in $\ln\left(\frac{L}{d}\right)$. The analysis of [6] gives a quantitative agreement with this behaviour. For moderate R_λ, d_m is too close to the dissipation scale for any reasonable linear range to develop. Only for atmospheric data can d_m be sufficiently large, thanks to large R_λ and a small turbulence ratio. For identical R_λ, jets flows have more fluctuations about the mean than wind tunnel flows, which explains why the helium jet data give even poorer results here, when using $C_{\ell\ell}$. However, agreement is found with wind tunnel when using C_2.

Finally, let us comment on noise in the helium data. Keeping this noise is a deliberate experimental choice, to avoid the mixing between noise and signal that would occur from filtering at lower frequencies. The noise is uncorrelated with the signal and its contribution to averaged quantities like moments of velocity increments can be evaluated and corrected. In [4], the same helium data used in [1] are shown to give good results for C_2 and μ.

Thus, using data analysis which is less influenced by Taylor hypothesis, one comes to the conclusion that the exponent μ does not depend on the Reynolds number.

[1] J. Cleve, M. Greiner, B. R. Pearson, and K. R. Sreenivasan, Phys. Rev. E **69**, 066316 (2004).
[2] B. Dhruva, Ph. D. thesis, Yale University, 2000.
[3] B. R. Pearson, P. A. Krogstad, and W. van de Water, Phys. Fluids **14**, 1288 (2002).
[4] O. Chanal, B. Chabaud, B. Castaing, and B. Hébral, Eur. Phys. J. B **17**, 309 (2000).
[5] J. Delour, J. F. Muzy, and A. Arnéodo, Eur. Phys. J. B **23**, 243 (2001).
[6] B. Castaing, Eur. Phys. J. B **29**, 357 (2002).
[7] A. N. Kolmogorov, J. Fluid Mech. **13**, 82 (1962).
[8] G. Stolovitzky, P. Kailasnath, and K. R. Sreenivasan, Phys. Rev. Lett. **69**, 1178 (1992).

